The mass of the charm quark from unquenched lattice QCD at fixed lattice spacing.

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We determine the mass of the charm quark (m_c) from lattice QCD with two flavors of dynamical quarks with a mass around the strange quark. We compare this to a determination in quenched QCD which has the same lattice spacing (0.1 fm). We investigate different formulations of the quark mass, based on the Vector Ward Identity, PCAC relation and the FNAL heavy quark formalism. Based on these preliminary results we find no effects due to sea quarks with a mass around strange.

1. INTRODUCTION

Quark masses are fundamental parameters of the Standard Model but due to confinement they cannot be measured directly by experiment and therefore any mass quoted always depends on the scheme (and scale) in which it was computed. The present world average value for $m_c^{\overline{\rm MS}}(m_c)$ has an error of approximately 17% [1]. This should be compared to the value quoted for the experimental mass of the D meson which is accurate to within less than 0.1%.

The mass of the charm quark is a difficult quantity to compute since it is light enough to be challenging for heavy quark methods and yet heavy enough such that the lattice spacing may be too coarse for the Compton wavelength. There has recently been several calculations of this quantity from the lattice [2,3,4,5]. In particular, a detailed study by Rolf and Sint [3] has been presented in the continuum limit, thus eliminating the error arising from lattice artifacts. However, the effects of including quark loops within these calculations has so far been omitted. This work is an attempt to estimate these effects and obtain a result in full QCD (albeit at fixed lattice spacing). In the interest of reducing errors resulting from a finite lattice spacing, we investigate several different formulations of m_c on the lattice.

We have previously published results from this data set on the hyperfine and P-wave mass splittings [6]. The mass splittings between the P-wave and S-wave mesons have been compared to the mass of the $D^*_{sJ}(2317)^+$ meson recently discovered by BaBar.

2. DEFINING THE QUARK MASS ON THE LATTICE

In this section we outline the definitions of bare quark mass used in this study and discuss some aspects of the renormalisation and improvement procedures. We use three different definitions, the first of which is the vector Ward Identity (VWI), given by

$$m_V = \frac{1}{2}\left(\frac{1}{\kappa_h} - \frac{1}{\kappa_{\rm crit}}\right). \quad (1)$$

where κ is the hopping parameter, the subscript h denotes the heavy quark and the subscript crit denotes the value of κ which corresponds to zero quark mass.

The second definition arises from the PCAC relation (AVWI) and is given by

$$m_A^Q + m_A^q = \frac{\langle \sum_x \partial_4 A_4^I(x) P^\dagger(0)\rangle}{\langle \sum_x P(x) P^\dagger(0)\rangle} \qquad (2)$$

where Q and q label the heavy and light quark respectively, A is the local axial current and P is the pseudoscalar density. The axial current has been $\mathcal{O}(a)$ improved.

The scaling studies of Rolf and Sint [3] show that $\mathcal{O}(a^2)$ lattice artifacts of the non-perturbative renormalisation scheme can be large. As some of the required coefficients have not been computed non-perturbatively at the parameter values used in this study, we use the (boosted) perturbative values, thus the lattice artifacts are $\mathcal{O}(\alpha_s a)$. As the lattice spacing is relatively coarse, we utilise an alternative definition of m_c. This is obtained by using the bare value obtained from the VWI, which is then identified as the "rest mass" in the FNAL formalism [7]. At tree level, this is defined as

$$am_1 = \log(1 + am_V). \qquad (3)$$

We use the one loop expression to connect the vector mass to the quark mass from Ref. [8]. In this work we use the rest mass of the hadron to compare with experiment, in the FNAL formalism the kinetic mass of the hadron is used. This is planned in future work.

Matching onto the $\overline{\mathrm{MS}}$ scheme is performed at one loop in perturbation theory for all definitions of the quark mass at the scale $q = 1/a$ and the associated systematic uncertainty is estimated by matching at $q = \pi/a$.

3. DETAILS OF THE COMPUTATION

3.1. Lattice specifications

There are four ensembles of gauge configurations available for computing the meson correlation functions but here we present details and results for the dynamical and coarsest quenched set only. The dynamical ensemble was generated with two degenerate flavors of sea quarks with a mass around the strange quark. Furthermore, the lattice spacing for the dynamical set was matched to that of the coarsest quenched ensemble (presented here) to facilitate the study of effects due to sea quarks without ambiguities arising from different lattice spacing errors. All ensembles were generated using a Wilson gauge action and the non-perturbatively $\mathcal{O}(a)$ improved fermion action was used to generate the quark propagators. These details are summarised in Table 1 and further details of the procedures for generating the dynamical ensemble and matching to the quenched data set can be found in Ref. [9].

Table 1
Lattice parameters

$(\beta, \kappa_{\mathrm{sea}})$	volume	a^{-1} (GeV)	N_{configs}
(5.93,0)	$16^3 \times 32$	$1.664(^{21}_{24})$	347
(5.2,0.1350)	$16^3 \times 32$	1.716(44)	395

The scale has been set using the "Method of Planes" technique which corresponds to setting $r_0 = 0.55\,\mathrm{fm}$ as the central value. The associated systematic error is then estimated using $r_0 = 0.5\,\mathrm{fm}$.

3.2. General approach

Having computed the correlators, the meson mass is obtained as a function of quark mass as follows. We fit to a 2 by 2 matrix of correlators (elements of which are combinations of local and fuzzed operators) and the fit region is obtained by studying the effective mass plots and monitoring χ^2/dof.

By extrapolating (interpolating) m_{light} to m_u (m_s), the D and D_s meson masses are obtained as functions of m_{heavy}. We then plot the data using the following interpolation formula

$$aM_H = a_0 + a_1 m_{\mathrm{heavy}} \qquad (4)$$

The experimental results for M_D and M_{D_s} are then used to read off the bare value of m_c.

4. PRELIMINARY RESULTS

Table 2 contains preliminary results for $m_c(m_c)$ in the $\overline{\mathrm{MS}}$ scheme for the quenched and partially quenched sets with the same lattice spacing. The FNAL result presented here is obtained from the rest mass definition and the errors quoted are statistical and systematic respectively. These results have been converted into Renormalisation Group Invariant (RGI) masses and plotted against the lattice spacing in Fig. 1 with some previous results shown for comparison.

Table 2
Preliminary results for $m_c^{\overline{\rm MS}}(m_c)$.

definition	NF	$m_c(m_c)$ (GeV)
AVWI	2	1.705(5)(115)
VWI	2	0.905(5)(130)
FNAL	2	1.228(3)(120)
AVWI	0	1.606(4) (96)
VWI	0	0.864(5) (95)
FNAL	0	1.261(4) (95)

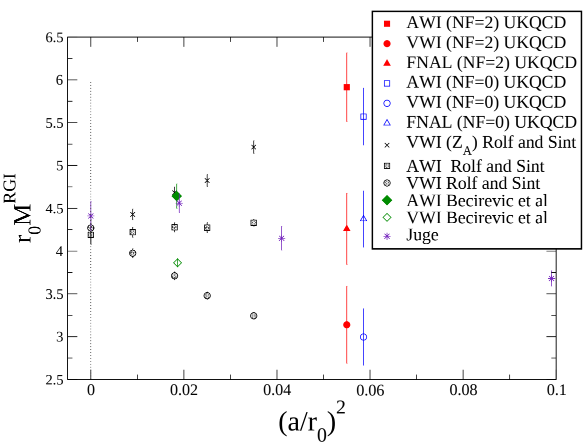

Figure 1. Recent data for the charm mass in the RGI scheme as a function of the lattice spacing.

5. CONCLUSIONS

There is no clear evidence of the effects of the sea quarks on the mass of the charm quark. This is almost certainly due to the unphysically heavy sea quark masses. However, large systematic uncertainties make it difficult to compare the quenched and dynamical results. One major hurdle in reducing the systematic error is the size of the lattice spacing. The dynamical results have been obtained at a fixed lattice spacing which is coarse. In comparison with the quenched case, it is very expensive to reduce the spacing in unquenched simulations and therefore we are unable to take the continuum limit which would enable us to control the systematic error. In response to this, we are investigating which formulation of m_c on the lattice minimizes discretization errors.

The mass independent renormalisation scheme suffers from large lattice artefacts of $\mathcal{O}((am)^2)$ for quark masses at coarse lattice spacings [3]. Some of the coefficients required for this scheme have large $\mathcal{O}(a)$ ambiguities themselves. Moreover, not all of these coefficients are known non-perturbatively for the dynamical case and so we have resorted to one-loop perturbation theory in this scheme. The only way to control errors further is to take the continuum limit which, as discussed, is unfeasible with the current generation of computers. An alternative approach is that of heavy quark methods such as the FNAL formalism, which removes mass dependent lattice artefacts of $\mathcal{O}(\alpha_s(am)^n)$. With careful application of this formalism, we may be able to make a comparison between the quenched and dynamical case. This is the subject of future work, as well as taking the continuum limit in the quenched case to control remaining lattice artefacts.

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